Lecture notes on risk management, public policy, and the financial system

Market equilibrium and relative risk

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Portfolios and investment choices

- Available investment choices can be expanded by mixing assets in **portfolios**
- Simple approach to identifying available investment choices
  - What combinations of **portfolio expected return** and **portfolio return variance** or **volatility**—representing risk—are available?
  - Are any of these combinations clearly superior or inferior to others?
- Based on expected returns, volatilities and correlations of constituent assets
- **Example:** Meta Inc. (ticker META) and Coca-Cola Co. (KO) 18May2012 to 24Sep2020

	META	KO
Mean daily logarithmic return (%)	0.090177	0.012446
Standard deviation of daily returns (%)	2.346570	1.141170
Correlation coefficient	0.21290	

- Once we understand menu of available and reasonable choices clearly, we can analyze which ones investors *prefer*

Portfolio expected return

- The **portfolio expected return** is a simple weighted average of the constituent assets' expected returns:
- In the case of just two constituents:

$$\mu_p = w\mu_1 + (1 - w)\mu_2,$$

  with $w \equiv$ asset 1 weight
- Portfolio expected return changes proportionally to a change in constituent expected return
- **Example:** the expected return of a 50-50 META-KO portfolio is

$$\mu_p = 0.5 \cdot 0.000902 + 0.5 \cdot 0.000124 = 0.00051311$$

  or 5.13 basis points per day

Portfolio return variance and volatility

- The **portfolio return variance** is *not* a weighted average of the constituent variances:

$$\sigma_p^2 = w^2\sigma_1^2 + (1-w)^2\sigma_2^2 + 2w(1-w)\sigma_1\sigma_2\rho_{12},$$

  - **Portfolio volatility** is its square root: $\sigma_p = \sqrt{\sigma_p^2}$
- Portfolio variance and volatility do *not* change proportionally to a change in constituent volatility
- And the portfolio variance can be strongly influenced up or down by return correlation
- **Example:** the return variance of a 50-50 META-KO portfolio is

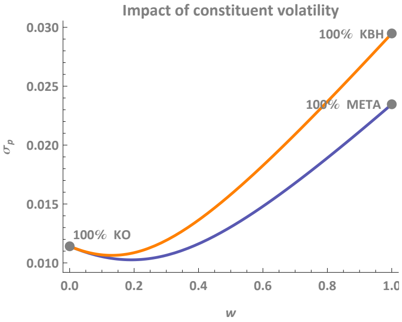
$$\begin{aligned}\sigma_p^2 &= 0.5^2 \cdot 0.000551 + 0.5^2 \cdot 0.000130 \\ &\quad + 2 \cdot 0.5 \cdot 0.5 \cdot 0.0235 \cdot 0.0114 \cdot 0.2129 = 0.000199\end{aligned}$$

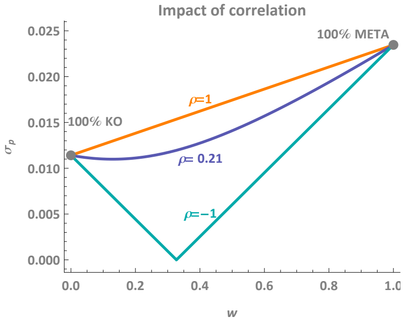
and the portfolio volatility is $\sqrt{0.000199} = 0.01409690$ or 1.410 percent daily.

Diversification is powerful

- **Diversification:** combining assets can lead to a reduction of risk without sacrificing return
- Diversification expands investors' opportunity set
  - Adding a small amount of even a high-volatility asset can reduce portfolio volatility
  - But effect more limited if return correlation strongly positive
- Lower correlation enables investor to achieve lower portfolio volatility for any given expected return
  - Negative correlation provides the strongest volatility reduction
  - Mixing risky assets can reduce portfolio return volatility even if correlation is positive

Impact of diversification on portfolio return volatility





Left panel: volatility (y-axis) of portfolios combining long positions in KO stock with long positions in META and KB Home (KBH), assuming a daily return correlation of 0, both plotted as a function of the META or KBH portfolio weight (x-axis). Right panel: volatility of portfolios combining long positions in KO stock with long positions in META assuming different non-zero return correlations.

Feasible and efficient portfolios

- Not every **feasible** or **attainable** portfolio is **efficient**
  - For each portfolio, find return and volatility
  - Two portfolios may have same volatility but different returns (or v.v.)
  - Portfolio with same volatility but lower return—or same return but higher volatility—than some other is not efficient
- **Efficient frontier:** return and volatility points of efficient portfolios
  - Traces risk-return tradeoff in mean-variance framework
- **Global minimum variance portfolio** has lowest return and volatility among efficient portfolios
- ($\rightarrow$)**Risk-free assets** may also be available for inclusion
  - Have non-zero return (usually but not always positive) but zero volatility

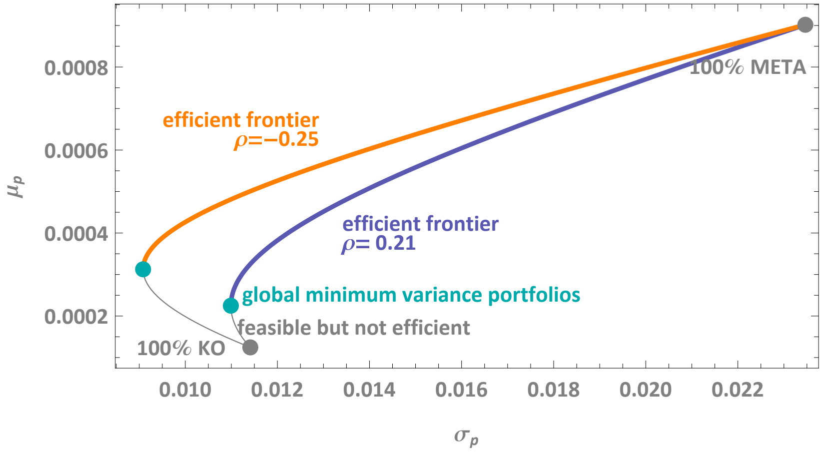
Feasible and efficient portfolios: example

- Portfolio consisting of all or mostly low-return/low-volatility KO not efficient
- Adding *some* high-return/high-volatility META lowers portfolio volatility and raises portfolio return
  - Unambiguously more desirable to investors than KO alone
- Portfolios with high share of META have higher volatility and return
  - May be more desirable to some investors

META wt. (%)	KO wt. (%)	return (%)	volatility (%)
0	100	0.012446	1.141170
10	90	0.020219	1.101140
12.92	87.08	0.022486	1.098950
20	80	0.027992	1.111820
50	50	0.051311	1.409690
90	10	0.082404	2.139120
100	0	0.090177	2.346570

In percent. The global minimum variance portfolio is highlighted.

The risk-return tradeoff



Volatility (x-axis) and mean (y-axis) of portfolios combining long positions in KO and META stock. Purple plot shows feasible portfolios estimated using the historical return correlation of 0.2129 . The heavy part of the plot is the efficient frontier. Orange plot shows efficient frontier if the return correlation were -0.25.

Explaining equilibrium asset prices and returns

- Market-clearing process determines asset prices and prospective returns by finding **equilibrium price**, given supply and demand schedules for securities
  - Assumptions about investors determine demand schedules
- Steps in the explanation:
  1. Take investment choices/prospective returns as given, analyze from individual point of view:
     1.1 Identify **efficient portfolios:** portfolios that don't waste opportunities
     1.2 Explain how individuals choose among efficient portfolios
  2. Once we know how individuals choose, how does market clear and establish the prospective returns individuals face?
- **Mean-variance framework:** payoffs on individual risky asset depend only on mean return and volatility

Investor preferences and risk

- Problems for quantitative definition of risk arise from preferences as well
- Mathematical optimization requires unambiguous preference ranking of sets of choices, portfolios
- Even with well-defined probability distribution of outcomes, difficulties in obtaining
  - Unambiguous preference ranking
  - Useful definition of **risk aversion**
- **Expected utility** axioms: require specification of **utility function**
- Approaches include

  Mean-variance dominance: provides limited ability to rank outcomes, doesn't consider tail returns

  Stochastic dominance looks at entire probability distribution

  - Approaches may contradict one another and may fail to provide unambiguous ranking

Choosing among portfolios

- Simple model: investor assumed to engage in **mean-variance optimization**
  - Happiness/wealth/utility increases with mean return and decreases with return volatility
- Modeled via utility function

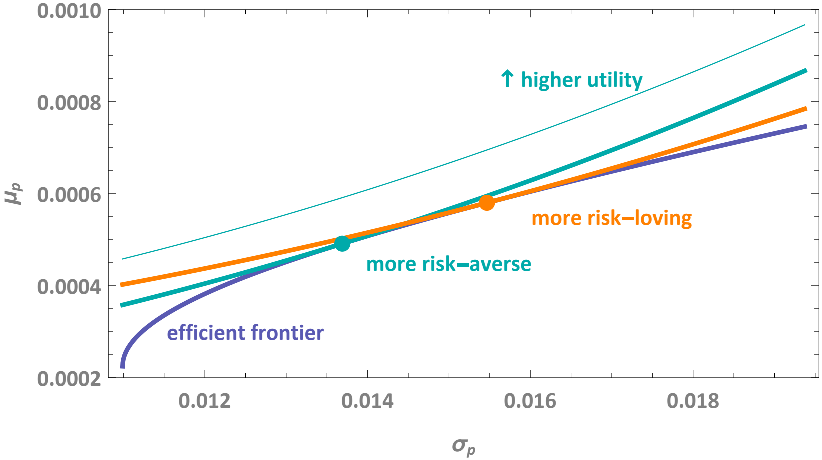
$$V(\mu_p, \sigma_p) = \mu_p - \frac{1}{2} k \sigma_p^2,$$

  with k expressing strength of investor's risk aversion
- **Indifference curves** express mean/volatility tradeoff
  - Defined by fixing utility at V° and differentiating the utility function

$$\left.\frac{d\mu_p}{d\sigma_p}\right|_{V=V^\circ} = k\sigma_p$$

  - The slope is positive: investor must be compensated with additional expected return if risk increases
  - Convex to the origin: slope is increasing in σ_p
- Investor chooses efficient portfolio that just touches the highest indifference curve she can achieve

Optimal investor choice among portfolios



Indifference curves for utility function $V(\mu_p, \sigma_p) = \mu_p - \frac{1}{2}k\sigma_p^2$ with $k = 4$ and $k = 2$ and efficient frontier of portfolios combining long positions in KO stock and META stock.

Investor choice if there is a risk-free asset

- Suppose there really were a risk-free security with certain return r^f
    - Its mean would also be r^f and its volatility zero
- Suppose investor able to lend or borrow freely at risk-free rate
    - Lending: invest in risk-free asset
    - Borrowing: finance additional risky assets ($\rightarrow$leverage)
- We can then define

  Expected excess return of an asset: the difference $\mu_i - r^f$ between its expected return and the risk-free rate

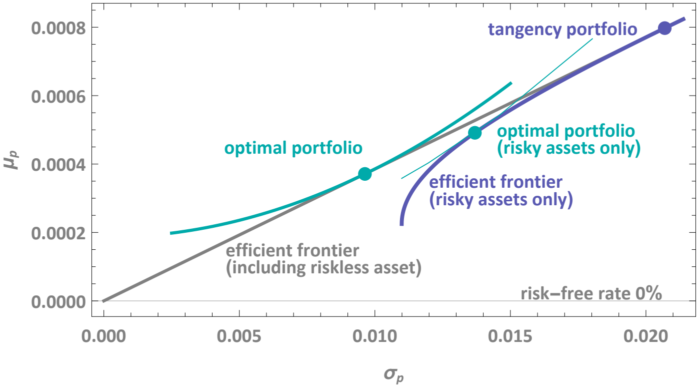
  Sharpe ratio of an asset: ratio $\frac{\mu_i - r^f}{\sigma_i}$ of excess return to volatility

    - Expected excess return per unit of risk
    - Reported Sharpe ratios usually *ex post*, based on realized/historical estimate of expected future return

Two-fund separation

- Also called **mutual fund theorem**
- All investors have same risky asset portfolio but different amounts of risk-free asset and risky portfolio
  - Risky asset portfolio has same constituents and same weights within the portfolio for everyone
- What is that risky asset portfolio?
- If there is a risk-free asset, efficient frontier$\rightarrow$ray from $(0, r^f)$ through **tangency portfolio**
  - Tangency portfolio is risky asset portfolio common to all investors
  - Tangency$\Rightarrow$attainable risky asset portfolio with highest Sharpe ratio
  - $\Rightarrow$Slope of efficient frontier is highest attainable Sharpe ratio
- Investor mixes risk-free asset and risky portfolio
  - The mix depends on her risk preferences

Optimal investor choice with a risk-free asset



Efficient frontier of portfolios combining only long positions in KO stock and META stock, efficient frontier of portfolios that also include a risk-free asset, with $r^f = 0$, and the indifference curve for $V(\mu_p, \sigma_p) = \mu_p - \frac{1}{2}k\sigma_p^2$ with $k = 4$ at the optimal portfolio of a risk-free as well as risky assets. The **tangency portfolio** is the point on the efficient frontier of portfolios combining risky assets only that is tangent to the efficient frontier, a line through $(0, r^f)$.

Summary of optimal investor choice

Weight	No risk-free asset		Including risk-free asset		
	$k=3$	$k=4$	tangency	$k=3$	$k=4$
Risk-free	NA	NA	NA	0.379	0.535
META	0.586	0.472	0.866	0.537	0.403
KO	0.414	0.528	0.134	0.083	0.063

Weights in the optimal portfolio of a mean-variance investor with utility function $V(\mu_p, \sigma_p) = \mu_p - \frac{1}{2}k\sigma_p^2$ for $k = 3, 4$. Optimization is over portfolios combining only long positions in KO stock and META stock, or portfolios that also include a risk-free asset.

Risk premiums and equilibrium market prices

- Risky asset prices embed **risk premiums**:
  - *Expected* excess return $\mu_i - r^f$ of risky (r_i) over risk-free security
  - Market discounts risky income streams at risk-free rate *plus* risk premium
- Investment i's Sharpe ratio $\frac{\mu_i - r^f}{\sigma_i}$ is the ratio of risk premium to volatility
- Risk premiums not directly observable, must be estimated via model of how market finds equilibrium asset prices
  - Equilibrium prices then related to investor preferences as well as assets' return characteristics
  - Leads also to explanation of relative prices of different risky assets

Overview

- **Capital asset pricing model** (CAPM):
  - All risk premiums driven by risk appetites and common source of risk
- Risk premium of any security is compensation for **systematic risk**
  - Related to risk of **value-weighted market portfolio** of all risky assets
  - Obviates need for or validity of security-specific analysis ($\rightarrow$efficient markets)
- Diversification$\rightarrow$shedding *uncompensated* **nonsystematic** or **idiosyncratic risk**
- CAPM a model of relative rather than absolute risk
  - CAPM does not itself provide measure of risk of market portfolio, i.e. systemic risk
  - $\Rightarrow$Volatility estimation

CAPM assumptions

- Agents are all mean-variance optimizers, and don't care about other distributional characteristics
- Complete information and agreement on means and variances of uncertain security returns
- Agents are not, however, identical
  - But they may have different risk preferences/aversion:⇔differ in their pricing of risk
- Market portfolio well-defined, has identifiable observable counterpart, non-traded assets unimportant
  - Conventionally proxied by broad stock index, e.g. S&P 500, the observable **market factor**
- Market clearing with no frictions
- There is a risk-free asset at which all agents can freely borrow or lend
  - Risk-free rate typically proxied by U.S. Treasury bill yield or return

Key results of the CAPM

- The market portfolio is an efficient portfolio
  - Since all investors agree on expected return and volatility of each asset, each chooses the same portfolio of risky assets
  - All markets clear, so all investors must be choosing market portfolio to combine with risk-free asset
- **Mutual fund theorem:** all investors will engage in two-fund separation
  - Each will choose a mix of the market portfolio and the risk-free asset depending on her own risk preferences
- The market portfolio is the only source of risk
  - CAPM consistent with single risk premium "risk-on/risk-off" world

The CAPM beta

- CAPM a model of prices and risk relative to market portfolio
  - Any specific asset i's risk premium related via **beta** to co-movement with that of market portfolio $\mu_m - r^f$

$$\mu_i - r^f = \beta_i(\mu_m - r^f)$$

  - Specific securities thereby priced relative to one another
- Where's α? It's *zero* in the CAPM
- An asset i's beta can be calculated from its excess return volatility σ_i, that of the market portfolio σ_m, and their correlation $\rho_{i,m}$:

$$\beta_i = \rho_{i,m}\frac{\sigma_i}{\sigma_m}$$

  - Beta increasing in correlation *and* asset volatility

The CAPM and risk premiums in equilibrium

- Any asset's Sharpe ratio related to that of the market portfolio by

$$\frac{\mu_i - r^f}{\sigma_i} = \rho_{i,m} \frac{\mu_m - r^f}{\sigma_m}$$

- Since $\rho \leq 1$, no asset can have a higher Sharpe ratio in equilibrium than the market portfolio

Systematic and nonsystematic risk

- Asset i's excess return variance σ_i^2 a measure of its risk
- Market portfolio's excess return variance σ_m^2 a measure of market risk
    - Beta captures comovement with/risk sensitivity to market factor
- σ_i^2 can be decomposed into
  Systematic risk: $\beta_i^2\sigma_m^2$, the part of σ_i^2 (or σ_i) related to fluctuations in market returns
  Idiosyncratic or **nonsystematic risk:** the remainder $\sigma_i^2 - \beta_i^2\sigma_m^2$, due to vagaries of individual firm's returns alone
- Can be expressed as shares of total

$$\frac{\beta_i^2\sigma_m^2}{\sigma_i^2} + \frac{\sigma_i^2 - \beta_i^2\sigma_m^2}{\sigma_i^2} = 1$$

- Systematic risk share in terms of excess return correlation:

$$\frac{\beta_i^2\sigma_m^2}{\sigma_i^2} = \rho_{i,m}^2 \frac{\sigma_i^2}{\sigma_m^2}\frac{\sigma_m^2}{\sigma_i^2} = \rho_{i,m}^2$$

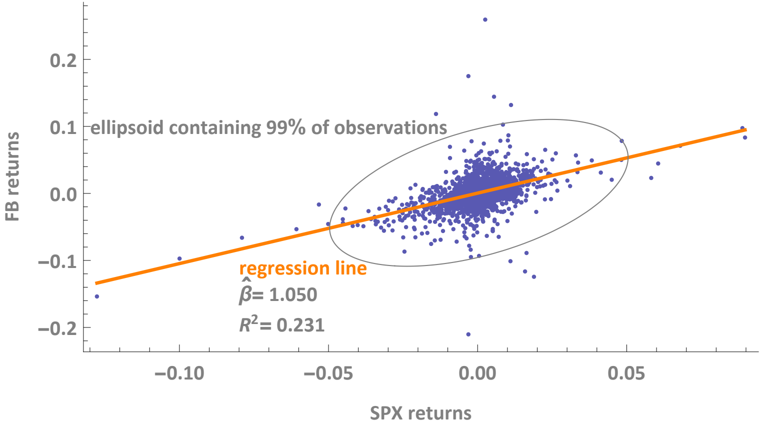
Computing the CAPM beta

- Based on simple **linear regression** model of security i's excess returns on market portfolio's excess return

$$r_{it} - r_{ft} = \alpha_i + \beta_i(r_{mt} - r_{ft}) + u_{it}, \qquad t = 1, \ldots, T$$

  - u_{it} assumed i.i.d. or normal, and independent of $r_{mt} - r_{ft}$
  - Daily, weekly or monthly observations
- Leads to estimates $\hat{\alpha}_i$ and $\hat{\beta}_i$
  - The CAPM model predicts $\alpha_i = 0$
- $\hat{\sigma}_{u,i}$ is the **residual mean square** or **standard error of the regression**
  - $\hat{\sigma}^2_{u,i}$ an unbiased estimate of variance $\sigma^2_{u,i}$ of the model error term u_i

Computing beta: an example



Computation of beta of the META to S&P 500, using 2087 unweighted daily excess return observations 18May2012 to 24Sep2020, relative to 3-month U.S. T-bill yield at the beginning of the return period. Points mark daily excess return pairs, expressed as decimals.

Estimating systematic and nonsystematic risk

- Interpret standard regression properties in context of CAPM
- Decompose stock i's observed excess return variance $\frac{\sum_t (r_{it} - r_{ft})^2}{T-1} = \hat{\sigma}_i^2$ into
  - Explained and residual variance
  - $\rightarrow$Systematic and nonsystematic risk
- Explained or systematic variance can be expressed as

$$\hat{\beta}_i^2 \frac{\sum_t (r_{mt} - r_{ft})^2}{T-1} = \hat{\beta}_i^2 \hat{\sigma}_m^2 = R^2 \hat{\sigma}_i^2$$

  - The estimated $\hat{\beta}_i^2$ times the variation in market excess returns
  - The estimated **coefficient of determination** (unadjusted) R^2 times the variation in stock i's excess returns
  - R^2 equals sample excess return correlation
- Residual or nonsystematic variance is the difference:

$$\hat{\sigma}_i^2 - \hat{\beta}_i^2 \hat{\sigma}_m^2 = \frac{T-2}{T-1} \hat{\sigma}_{u,i}^2$$

Example: systematic and nonsystematic risk

	META	KO	KBH
Parameter estimates:			
$\hat{\alpha}_i$	0.0004435	-0.0001763	0.0001699
$\hat{\beta}_i$	1.051	0.674	1.456
Goodness-of-fit and correlation:			
R^2 (unadjusted)	0.23068	0.40096	0.28040
Adjusted R^2	0.23032	0.40067	0.28006
Excess return correlation to S&P $\hat{\rho}_{i,m}$	0.21293	0.22568	0.48030
Risk decomposition:			
Variance of excess returns $\hat{\sigma}_i^2$	0.0005507	0.0001302	0.0008689
Systematic variance $\hat{\beta}_i^2\hat{\sigma}_m^2$	0.0001270	0.0000522	0.0002436
Nonsystematic variance $\hat{\sigma}_i^2 - \hat{\beta}_i^2\hat{\sigma}_m^2$	0.0004236	0.0000780	0.0006253
Risk decomposition (share of total variance):			
Systematic variance	0.23068	0.40096	0.28040
Nonsystematic variance	0.76932	0.59904	0.71960

Excess returns relative to 3-month U.S. T-bill yield.

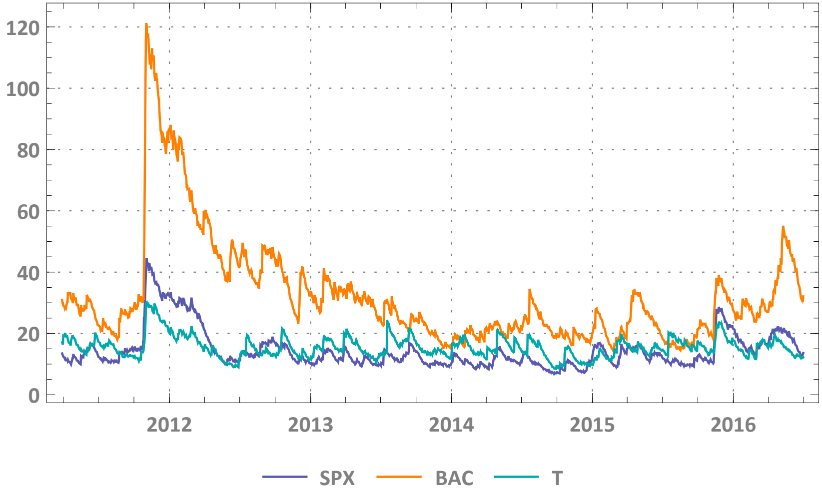
Beta, correlation, and volatility

- Excess returns of high-beta stocks typically, but not always strongly correlated with market's
- Relationships among beta, correlation, and market and asset volatilities constrained by

$$-1 \leq \beta_i \frac{\sigma_m}{\sigma_i} = \rho_{i,m} \leq 1$$

- If high-beta returns much more volatile than market returns, correlation may be weak
  - $\Leftrightarrow$Systematic risk share low, in spite of high beta
- High beta associated with higher return volatility than market, but tracking overall behavior of market return volatility
- **Examples:**
  - BAC has beta nearly triple that of T, but only moderately higher excess return correlation to the market
  - BAC has high beta, but systematic risk just a bit over $\frac{1}{2}$ of total

Volatility behavior of high- and low-beta stocks



Annualized EWMA volatility for BAC, T and the S&P 500, daily 03Jan2011 to 30Jun2016.

Empirical validation of the CAPM

- CAPM implies **cross-sectional variation** across individual stocks in *expected* returns at a point in time fully explained by beta
  - Expected returns measured empirically via realized excess returns
- In tests, CAPM does not fully capture systematic influences on individual stock prices
- CAPM a **single-factor** or **single-index** model
  - $\Rightarrow$Search for additional explanatory/priced factors ("smart beta")
- **Fama-French three-factor model** includes in addition to market factor

  Small Minus Big (SMB): average return on small-cap minus return on large-cap portfolios

  High Minus Low (HML): average return on value (high **book-to-market**/low **price-to-book ratio**) minus return on growth (low book-to-market) portfolios
- **Momentum** factor: stocks with high recent returns

Limitations of the market proxy

- **Roll critique** or **market proxy problem**
- Validation of CAPM requires accurate identification of market portfolio
- Conventional proxies, e.g. S&P 500 index omit important elements of wealth, esp. human capital, non-U.S. assets
- Analogous to the ($\rightarrow$)**joint hypothesis problem** in testing market efficiency
    - Is the model or the proxy wrong?

Consumption CAPM

- Simplicity of the mean-variance optimization model contributes to empirical shortcomings of CAPM
- Investors care about many things, e.g.
  - Do high payoffs occur in good times or in bad, when they are more valuable?
  - Do high payoffs occur when investment opportunities are good, or capital goods cheap relative to consumption goods?
  - Tail risks, **rare consumption disasters**, e.g. financial crises and wars
- **Consumption CAPM:** asset prices driven by risk appetite, covariance of return with utility of consumption across states
  - Multiple periods, not just "now" and "future"
  - Declining **marginal utility of consumption:** additional consumption less valuable at higher consumption level
  - $\Rightarrow$Low asset payoffs in bad times less valuable to risk-averse agents ("anti-insurance"), lead to lower asset price/higher risk premium
  - $\Leftrightarrow$High payoffs in bad times$\rightarrow$higher asset price/lower risk premium

Stochastic discount factor and asset prices

- **Stochastic discount factor:** (SDF) discounted value of marginal utility of consumption, captures
  - **Time preference:** near-term more valuable than future consumption
  - **Risk preference:** how fast does marginal utility of consumption decline?
- SDF the same (for a particular or representative agent) for all assets
  - A different value of the SDF for each possible future state
- But state-contingent payoffs different for each asset
- Assets have positive risk premiums—are cheaper—if covariance of payoffs with SDF high

Risk factor approach to asset pricing

- Reduce dimensionality of covariance matrix of asset returns
  - Economic/market data that explain *variance* of returns
  - Possibly unobservable or **latent** characteristics
  - Definition of risk factors depends on model and available data
  - Asset returns as risk factors for other securities
- In efficient capital markets, risk premiums reflect **priced factors**
- $\rightarrow$**Arbitrage pricing theory** (APT) introduces multiple risk factors
  - Asset or portfolio returns accurately predicted by returns on a set of factors$\Rightarrow$portfolio can be replicated by the factors
  - **Example**: Fama-French model prices stocks more accurately than CAPM
  - Each factor carries with it a risk premium that compensates for low return just when you can least afford it
- Two-fold motivation of risk factor approach:

  Reality: many securities, far fewer meaningfully independent influences on them

  Parsimony: make high-dimensional problem tractable and intuitive

Portfolios and diversification

Investor choice

Models of market equilibrium: Capital Asset Pricing Model

Capital market efficiency

- Asset prices in an efficient market
- Validating the efficient markets hypothesis
- Efficacy of active management
- Behavioral finance

What are efficient markets?

- **Efficient markets hypothesis** (EMH) maintains that current asset prices reflect all currently available information
  - Absence or near-absence of arbitrage opportunities
- Different definitions of "all information"

  Weak-form EMH: all the information in past prices
  Strong-form EMH: all public and private information
- Asset prices are generally close to "correct" price
  - Variously described as **equilibrium price** or **fundamental value**
  - E.g. CAPM predicts $\alpha_i = 0$ *in equilibrium*
- **Rational expectations** (more or less):
  - Investors' subjective expected value of future asset values equal to best statistical estimate given available information set
  - But choices nonetheless determined by risk preferences and entire distribution of outcomes
- Markets are permitted to clear

Efficiency can only be approximated in the real world

When there's steak on the table, eat steak trading desk maxim

- How fast should market prices adjust and how close should they be to equilibrium to validate EMH?
  - Market process time-consuming, requires costly information-gathering
  - Equilibrium price itself not observable and may be changing over time
- **Limits to arbitrage** and **slow arbitrage**:
  - Position-taking requires costly information-gathering and time that must be compensated
  - ⇒Asset markets cannot be perfectly strong-form efficient (**Grossman and Stiglitz paradox**)
- Long-term reversion: "cheap" assets—with unusually low recent returns—often have high returns over subsequent several *years*
  - E.g. following overall market slumps, sold-off sectors and firms
- But prices act as signals even if not perfectly correct at any time
  - Tendency to equilibrium
  - But equilibrium never actually reached as world changes
- Fisher Black: efficiency if "price is within a factor of 2 of value."

Risk and market efficiency

- Most "arbitrage" opportunities involve some risk-taking
- Presence of time-varying **risk premiums** is
- Can be identified empirically using models, information variables related to risk that help predict returns
  - Excess returns then related to risk, not successful arbitrage
- But persistent riskless arbitrage opportunities *not* compatible with EMH

Empirical evidence on market efficiency

- Implications of market efficiency:
  - Changes in market prices not predictable (**random walk hypothesis**)
  - Impossibility of systematically earning "abnormal" returns—higher risk-adjusted returns than the market
- Empirical evidence on market efficiency drawn from several sources

  Time series behavior of asset returns: do past returns alone help predict future returns?

  Return forecasts based on information variables: do current variables other than past returns help predict future returns?
  - E.g. data on company fundamentals, economic data

  Efficacy of active management by mutual and hedge funds

  Event studies: surprises should have instantaneous but non-persistent impact on price
- Methodological difficulties introduced by trading costs, changes in risk and risk appetite over time

Efficiency and return time series behavior

- EMH implies past returns contain (close to) no information that could help forecast future returns
- **Law of iterated expectations:** knowledge grows over time
  - Future equilibrium price will be based on information available in the future
  - New information available in future is unknowable now
  - ⇒Today's estimate of future price equals that based on current information
  - ⇒Expected value of price return based on current information must be zero
- **Autocorrelation** or **serial correlation** of returns—correlation of returns in successive periods—near zero
  - Validates EMH: past returns contain almost no information that could help forecast future returns
- Returns in successive periods uncorrelated, but not independendent
  - But higher moments, e.g. volatility, exhibit serial correlation

Joint hypotheses in tests of market efficiency

- The **joint hypothesis problem:** any test of market efficiency is bundled with a model of asset price determination
  - Non-rejection$\Rightarrow$non-rejection of *both* efficiency and pricing model
  - Rejection$\Rightarrow$rejection of *either* efficiency or pricing model (or both)
- Finding abnormal returns to a strategy may just reflect inadequacies of model
- Tests of any pricing model are also tests of efficiency
  - E.g. CAPM valid only if markets efficient
- Risk premiums, equilibrium prices vary over time
  - Pure random walk hypothesis requires constant expected returns
  - Excess returns are risk premiums, returns to priced factors
- Failures of efficiency tests$\rightarrow$revised pricing models
  - Violation of variance bounds, predictive value of dividend yield: evidence of time-varying price of risk
  - Search for priced factors ("smart beta")

Active and passive investment management

Passive management: imprecise term used to describe range of investment strategies including

- Narrow definition: holding the value-weighted market portfolio ("the market") at all times
  - Strictly defined, encounters problems of defining market portfolio, e.g. human capital, other non-traded or illiquid assets
- Investing largely in value-weighted market portfolio but applying "tilts" based on multi-factor models
- Investing in index mutual funds or ETFs

Active management: deviating from passive management, based i.a. on

- Applying an investment strategy based on inefficiencies in capital markets
- Possession of information superior or more accurate than that incorporated into current asset prices
- Ability to identify asset managers capable of earning higher risk-adjusted excess return than the market

Does active management pay off for investors?

- Little evidence of “beating market,” adjusted for risk, trading costs
- Dimensions of predictive ability in active management

  Asset price forecasting or stock picking

  Market timing: forecasting the general return level of all risky assets
- Challenge of identifying persistence, luck/false positives
  - Can identifiable active managers predictably outperform over the long term?
- Skill-based excess returns accrue only to manager as fees rise and opportunities exploited
- Some managers may have skill, but investors lack ability to identify them *ex ante*
  - Managers may trade solely to suggest they have superior information
- Absence of successful trading rules, in spite of evidence of autocorrelation of functions of returns, long-term predictability
- Successful active management improves *risk-adjusted* returns

Measuring active management performance

- Framework: Compare active managers' monthly or annual returns to performance of model—set of benchmarks $\mathbf{x}$
  - E.g. CAPM or Fama-French model set benchmarks
- Via time series regression of j managers' returns on factor model

$$r_{jt} - r_{ft} = \alpha_j + \mathbf{x}_t \boldsymbol{\beta} + \varepsilon_{jt}, \qquad t = 1, \ldots, T$$

- $r_{jt} - r_{ft}$ measured **gross** or **net** of costs
  - Costs include administrative, salary and trading expenses of manager
  - But exclude load, other sales charges or commissions that further reduce investor return
- Result is a distribution of estimated $\hat{\alpha}_j$
  - Active manager out(under)performs$\rightarrow$ $\hat{\alpha}_j > 0 (< 0)$

Testing for active management outperformance

- Many active managers achieve superior returns, many underperform
    - Expect to observe $\hat{\alpha}_j \gtrless 0$ for most j—consistent with efficiency?
    - How discern if significant fraction of managers "beat the market"?
- t-statistic $t(\hat{\alpha}_j)$: normalize $\hat{\alpha}_j$ by its standard error $\sigma(\hat{\alpha}_j)$

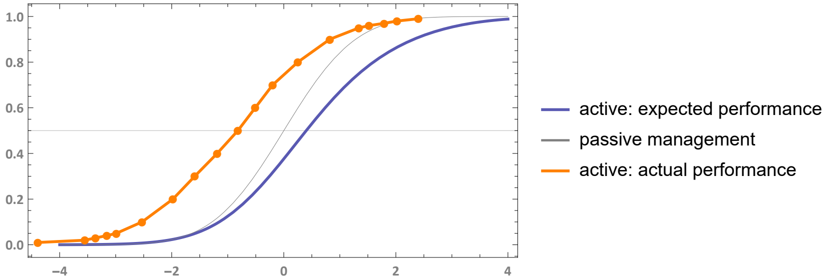
$$t(\hat{\alpha}_j) = \frac{\hat{\alpha}_j}{\sigma(\hat{\alpha}_j)}$$

- Active manager performance matches benchmark$\Rightarrow$ $t(\hat{\alpha}_j) \sim \mathcal{N}(0, 1)$
- Active manager outperformance$\Rightarrow$distribution of $t(\hat{\alpha}_j)$
    - Has mean > 0 $\Rightarrow$most active managers outperform
    - And/or skewed to positive values$\Rightarrow$significant fraction of managers outperform

Evidence on active management outperformance

- The $t(\hat{\alpha}_j)$ don't look like a sample from $\mathcal{N}(0,1)$
- Mean of distribution of $t(\hat{\alpha}_j)$ using gross returns close to zero
- Mean of distribution of $t(\hat{\alpha}_j)$ using net returns < 0
- Distribution of $t(\hat{\alpha}_j)$ using gross or net returns skewed to large *negative* values
- Interpretation:
  - Typical active manager barely recoups management costs, doesn't outperform market
  - Typical active manager underperforms market once management costs accounted for
  - Active managers perform worse than if performance relative to benchmarks were entirely random

Evidence on active management outperformance



Stylized representation of the empirical results in the Jensen (1968), Fama and French (2010) and other papers comparing active mutual fund returns to benchmark models. The plots represent the densities or empirical distributions of *t*-statistics of fund managers' returns. Returns are measured on the *x*-axis in standard deviations of managers' excess returns over returns on value-weighted market portfolio, with excess returns measured net of investment management costs. The purple plot displays the expected empirical result if active management results in outperformance: a positive mean or a skew to outperformance. The orange plot displays the actual empirical result: active management has a *negative* mean and a skew to surprisingly large *underperformance*. The vertical grid lines mark the means of the two distributions. The gray displays results for the benchmark—the indexers.

Individual active manager efficacy

- It takes a very, very long time to assess one manager's performance
- Claims by active asset managers that they can "beat the market" are weak
- Many years of observations required to establish individual active manager efficacy with high confidence
- Suppose a market index used as benchmark
  - Manager claim: "beating the market" by α, e.g. 2 percent annually

    $$y = \alpha + \beta x,$$

    with y, x the manager and benchmark excess return over risk-free
  - Square of **standard error** $s^2(\hat{\alpha})$ of $\hat{\alpha}$ in regression estimate:

    $$s^2(\hat{\alpha}) = (1 - \bar{R}^2) s^2(y) \left[\frac{1}{T} + \frac{\bar{x}^2}{\sum_t^T (x_t - \bar{x})^2} \right] = \frac{(1 - \bar{R}^2) s^2(y)}{T} \frac{1 + \bar{x}^2}{s^2(x)}$$

Validating individual manager efficacy: example

- Represent single manager by HFRX Global Hedge Fund Index
  - Using past 30 years of data (moments at monthly rate):

$\bar{R}^2$	0.5
$s^2(y)$	0.045
$\bar{x}^2$	0.006
$s^2(x)$	0.015

  - Individual active manager likely has higher $s^2(y)$
- Test hypothesis $\alpha = 3$ percent annually or 25 bps/month
  - Would barely cover typical activist fees
  - Against null hypothesis $\alpha = 0$
  - Assume T large, use normal distribution, solve for T:

$$1.96 = \frac{\hat{\alpha}}{s(\hat{\alpha})} = \frac{0.0025}{\sqrt{\frac{0.5 \times 0.05}{12T}\left(1 + \frac{0.006^2}{0.015^2}\right)}},$$

- 60 years needed for significance at 95 percent confidence level

Slow arbitrage and active management

- Some evidence that “patient capital” may earn excess returns
  - Institutional asset management rather than mutual funds
- Portfolios with significant differences from benchmarks and low turnover

Anomalies in asset prices and market behavior

Deviations of price from fundamental value: many securities, far fewer meaningfully independent influences on them

- **Data mining:** sifting through large amounts of data to find anomalies that pass significance tests

Excess volatility of asset prices compared to cash flows

- Source of volatility may be **time-varying** risk premiums related to macroeconomic conditions
- Model of risk, e.g. CAPM, required to adjudicate empirically

Large trading volumes: many securities, far fewer meaningfully independent influences on them

- **Noise trading** may be necessary for pricing and efficiency
- But in efficient market, how is anyone remunerated for the effort of information gathering?

Market efficiency and behavioral finance

- **Behavioral finance**: study of possible departures from efficiency attributable to non-rational behavior in
  - Evaluation of information, e.g. **overconfidence**
  - Ranking of outcomes, e.g. **loss aversion**
- Persistent anomalies in patterns of asset prices and market behavior interpreted as departures from rationality, evidence against efficiency
  - But anomalies not systematically exploitable$\Rightarrow$consistent with efficiency
- The clash of religions: excess volatility can be explained as result of
  - "Irrational" fluctuations in sentiment
  - Changes in risk appetites consistent with a reasonable posited utility function